Online Supplement: Robust Metric Inequalities for Network Loading under Demand Uncertainty

Grit Claßen[†], Arie M.C.A. Koster [*†], Manuel Kutschka[‡], and Issam Tahiri[††]

[†]RWTH Aachen University, Lehrstuhl II für Mathematik, Wüllnerstr. 5b, D-52062 Aachen, Germany, {classen, koster}@math2.rwth-aachen.de

[‡]INFORM GmbH, Aviation Division, Pascalstraße 35, 52076 Aachen, Germany, manuel.kutschka@inform-software.com

[††]Université Bordeaux, Institut de Mathématiques de Bordeaux, 351, cours de la Libération 33405 Talence cedex, France, issam.tahiri@math.u-bordeaux1.fr

A Appendix

	Γ	best PB	CPLEX	RCI	1-bounded RMI	rounded RMI	tight RMI
abilene1	0	621.4	462.6	582.6	582.6	582.6	582.6
	1	1126.4	979.2	1068.1	1087.5	1087.5	1087.5
	2	1126.4	1017.4	1106.9	1113.4	1113.4	1113.4
	3	1165.2	1044.4	1165.2	1165.2	1165.2	1165.2
	4	1165.2	1067.0	1165.2	1155.5	1155.5	1155.5
	5	1204.0	1086.5	1147.0	1165.2	1171.7	1165.2
	6	1204.0	1102.2	1165.2	1165.2	1171.7	1171.7
	7	1242.9	1114.5	1204.0	1204.0	1204.0	1204.0
	8	1242.9	1124.9	1223.5	1223.5	1223.5	1223.5
	9	1242.9	1132.8	1204.0	1213.8	1213.8	1213.8
	10	1242.9	1138.8	1204.0	1213.8	1213.8	1213.8

[*]corresponding author

abilene2	0	621.4	459.6	602.0	602.0	602.0	602.0
	1	660.3	525.9	602.0	621.4	621.4	621.4
	2	699.1	575.5	625.9	660.3	660.3	660.3
	3	699.1	609.7	660.3	686.2	699.1	699.1
	4	699.1	636.2	660.3	686.2	699.1	686.2
	5	738.0	657.8	669.4	738.0	718.5	719.5
	6	776.8	675.6	725.8	776.8	776.8	776.8
	7	776.8	692.4	745.0	776.8	776.8	776.8
	8	815.6	705.7	766.9	792.3	796.2	796.2
	9	854.5	716.1	798.8	815.6	807.9	808.2
	10	854.5	724.6	800.3	815.6	815.6	825.4
germany17	0	893.3	557.3	776.8	796.2	776.8	776.8
	1	1087.5	791.3	990.4	990.4	990.4	1009.8
	2	1165.2	894.2	1007.3	1029.3	1048.7	1074.7
	3	1242.9	950.4	1061.2	1097.2	1092.4	1161.7
	4	1242.9	992.7	1137.1	1145.8	1155.5	1192.9
	5	1281.7	1023.6	1168.6	1174.9	1175.4	1213.8
	6	1320.6	1050.3	1183.2	1193.5	1195.1	1253.0
	7	1359.4	1072.0	1193.6	1208.9	1209.3	1253.2
	8	1359.4	1089.7	1209.8	1233.2	1242.9	1262.7
	9	1359.4	1101.3	1225.0	1233.2	1233.2	1270.1
	10	1398.2	1110.3	1230.6	1267.2	1255.0	1324.4
geant1	0	971.0	521.1	762.2	757.4	757.4	757.4
	1	1009.8	629.9	848.0	873.9	873.9	878.2
	2	1087.5	692.4	912.7	932.2	912.7	932.2
	3	1126.4	728.8	926.8	941.9	961.3	965.5
	4	1126.4	756.2	937.5	951.6	951.6	985.8
	5	1126.4	775.9	943.7	980.7	971.0	1006.5
	6	1165.2	792.4	969.1	990.4	971.0	1008.0
	7	1165.2	805.2	960.7	994.3	978.8	1038.8
	8	1165.2	816.4	979.1	982.7	982.7	1010.7
	9	1165.2	826.1	964.6	1004.3	1004.3	1027.3
	10	1165.2	834.4	984.7	1013.7	989.1	1008.9
geant2	0	1009.8	492.9	738.0	776.8	776.8	776.8
	1	1087.5	663.4	932.2	932.2	932.2	948.3
	2	1165.2	763.9	998.6	1019.6	1019.6	1019.5
	3	1204.0	814.1	1029.3	1029.3	1029.3	1048.7
	4	1204.0	849.5	1052.9	1087.5	1087.5	1087.5
	5	1242.9	870.8	1078.8	1087.5	1077.8	1116.7
	6	1281.7	886.0	1075.0	1103.1	1095.3	1126.4
	7	1281.7	897.8	1089.2	1131.2	1131.2	1116.7
	8	1281.7	907.8	1096.0	1126.4	1113.4	1113.4
	9	1320.6	916.6	1106.9	1119.9	1126.4	1119.9
	10	1320.6	923.5	1106.9	1126.4	1139.3	1126.4

Table A.1: Best known primal bound and dual bounds of CPLEX, RCI, 1-bounded RMI, rounded RMI, and tight RMI at the root node for the compact flow formulation. Dual bounds marked by * are not necessarily optimal since the solving stopped at the time limit before the root node was completed.

		compact flow formulation						capacity formulation			
		solution time	speed-up factors								
	Γ	I: CPLEX [s]	IIs	IIc	IIIs	IIIc	IVc	Ic	IIc	IIIc	IVc
abilene1	0	0.9	5.00	0.63	0.56	0.50	0.50	0.56	2.50	1.25	0.83
	1	2.1	0.44	0.44	0.05	0.12	0.13	0.38	1.14	0.15	0.09
	2	2.4	0.56	0.71	0.56	0.71	0.77	0.42	1.25	0.08	0.08
	3	2.4	0.86	0.63	0.86	0.63	0.63	0.50	0.80	0.32	0.23
	4	3.1	0.79	1.00	0.79	1.00	1.00	0.35	0.65	0.14	0.13
	5	1.8	0.26	0.26	0.11	0.09	0.08	0.67	0.55	0.11	0.10
	6	2.3	0.68	0.50	0.68	0.50	0.50	0.65	1.07	0.05	0.05
	7	3.2	1.04	1.33	0.76	1.27	1.33	0.88	2.00	1.17	1.04
	8	2.8	1.88	0.97	1.88	0.63	0.63	1.07	2.31	1.30	0.88
	9	2.5	2.13	1.55	2.13	1.55	1.55	1.36	2.83	1.03	0.83
	10	2.1	0.76	0.68	0.76	0.35	0.26	0.90	1.12	0.27	0.31
abilene2	0	0.9	1.50	0.30	0.60	0.19	0.19	0.33	0.75	0.60	0.50
	1	1.7	0.57	0.80	0.55	0.20	0.20	0.94	1.60	0.24	0.21
	2	2.7	0.49	0.83	0.17	0.19	0.19	0.70	1.58	0.14	0.13
	3	2.8	0.57	0.59	0.54	0.57	0.39	0.46	0.93	0.13	0.16
	4	1.7	0.39	0.35	0.39	0.33	0.25	0.41	0.70	0.03	0.01
	5	3.1	0.20	0.25	0.09	0.15	0.29	0.26	0.25	0.01	0.02
	6	3.3	0.47	0.70	0.41	0.70	0.67	0.42	0.78	0.08	0.05
	7	3.2	1.33	1.60	1.33	1.60	1.60	0.25	0.29	0.01	0.02
	8	3.5	1.50	1.80	0.35	0.28	0.28	1.29	2.14	0.14	0.21
	9	5.2	0.97	1.43	0.05	0.32	0.30	0.58	1.43	0.07	0.05
	10	3.4	1.57	1.26	0.57	1.47	1.47	1.29	2.75	0.07	0.06
germany17	0	6.7	22.83	4.70	18.39	4.11	4.14	9.88	9.19	1.76	1.65
	1	25.1	6.15	3.96	6.12	3.44	3.48	5.29	28.26	11.35	10.71
	2	32.2	0.93	2.25	0.65	0.53	0.52	1.99	2.95	0.22	0.30
	3	58.6	5.52	10.38	3.67	4.54	4.59	8.16	9.32	3.56	3.59
	4	51.2	2.86	4.32	0.81	1.58	1.57	2.38	2.04	0.60	0.48
	5	42.7	3.84	3.62	1.62	1.49	2.53	5.70	1.82	0.66	1.19
	6	53.5	4.10	5.94	2.01	3.80	2.86	3.56	7.20	1.15	1.29
	7	91.8	4.86	2.30	4.40	2.76	2.78	5.26	10.87	2.47	3.55
	8	45.8	2.06	7.31	1.24	1.57	1.50	5.14	5.19	1.53	1.23
	9	33.6	3.13	1.36	1.13	1.00	0.98	6.23	5.17	1.57	1.70
	10	53.2	3.54	4.90	2.68	3.12	3.03	10.85	16.26	3.40	5.34
geant1	0	1640.8	6.59	8.39	6.54	8.33	8.47	5.73	57.26	57.33	47.89
	1	853.9	1.00	1.15	1.55	1.23	1.18	3.83	3.83	5.93	8.07
	2	1053.3	3.06	2.48	3.02	2.74	2.78	19.91	68.53	27.15	49.31
	3	3517.5	4.62	1.88	2.86	4.45	4.46	8.85	25.20	23.06	22.21
	4	2141.3	1.18	1.67	1.09	1.53	1.54	3.99	13.80	9.94	13.43
	5	1757.7	2.02	4.12	2.36	3.36	3.21	2.98	16.05	4.35	9.96
	6	4587.1	1.33	1.84	2.03	2.32	2.41	6.84	19.13	15.36	18.98
	7	3352.6	1.30	2.72	2.21	2.16	2.10	4.96	14.92	13.88	13.42
	8	2067.1	3.44	3.49	2.86	2.94	2.88	7.06	15.19	9.13	12.38
	9	1837.3	0.97	3.22	3.41	2.83	2.84	4.84	9.93	7.21	5.72
	10	1443.9	2.16	8.33	3.92	6.59	7.05	11.36	24.01	15.56	12.53
geant2	0	2244.6	3.17	1.37	3.29	2.35	2.40	19.25	68.10	73.08	68.98
	1	2571.1	1.95	3.28	1.96	3.12	3.19	3.29	9.35	6.47	9.26
	2	11821.2	1.52	1.74	0.80	1.19	1.21	0.47	0.90	0.89	0.88
	3	13299.4	0.62	0.49	0.60	0.49	0.51	0.47	0.77	0.73	0.59
	4	6084.4	1.42	3.17	2.39	3.27	3.34	0.48	1.85	2.32	0.85
	5	6699.6	2.43	3.56	1.32	1.56	1.54	0.58	1.07	0.79	0.69
	6	16436.9	1.80	1.81	2.28	1.82	1.86	0.45	1.42	1.43	1.47
	7	8962	1.57	1.44	1.69	1.52	1.54	0.44	1.53	0.96	0.97
	8	3792.4	1.27	0.52	1.27	0.50	0.52	1.31	2.23	1.53	1.98
	9	18969.8	5.97	3.27	6.21	3.15	3.65	1.94	6.90	6.75	7.33
	10	13286.4	5.61	1.34	5.55	1.32	1.33	1.26	5.26	4.54	5.05

Table A.2: Speed-up factors of algorithms Ic, IIs, IIIs, IIc-IVc compared to the solution times of CPLEX and the compact flow formulation of the robust network loading problem.

		I: compact		Ic: capacity		IIc: capacity		IVc: capacity	
	Γ	PB	DB	PB	DB	PB	DB	PB	DB
germany50	0	2019.7	1351.0	1980.8	1299.0	1980.8	1472.7	1980.8	1472.6
	1	2213.9	1427.6	2136.2	1396.7	2213.9	1587.8	2213.9	1582.7
	2	2213.9	1523.5	2291.6	1262.6	2291.6	1630.1	2291.6	1628.1
	3	2252.7	1571.7	2408.1	1165.5	2408.1	1649.2	2408.1	1649.2
	4	2291.6	1597.4	2446.9	1223.8	2485.8	1671.7	2485.8	1674.7
	5	2369.2	1629.5	2408.1	1165.2	–	1671.3	–	1672.5
	6	2408.1	1645.6	2446.9	1223.5	2485.8	1705.1	2485.8	1707.7
	7	2369.2	1683.2	2563.4	1204.0	2524.6	1709.0	2524.6	1709.0
	8	2291.6	1701.2	2524.6	1204.1	–	1728.4	–	1727.5
	9	2408.1	1714.8	2446.9	1179.9	2524.6	1747.8	–	1738.1
	10	2485.8	1740.0	2602.3	1205.4	2680.0	1766.4	2680.0	1766.7

Table A.3: Primal and dual bounds of algorithms Ic, IIc, and IVc for the capacity formulation compared to the bounds of CPLEX with the compact flow formulation of the robust network loading problem.