Algorithm Graph Theory: How hard is your combinatorial optimization problem?

Short Course – Lecture 4

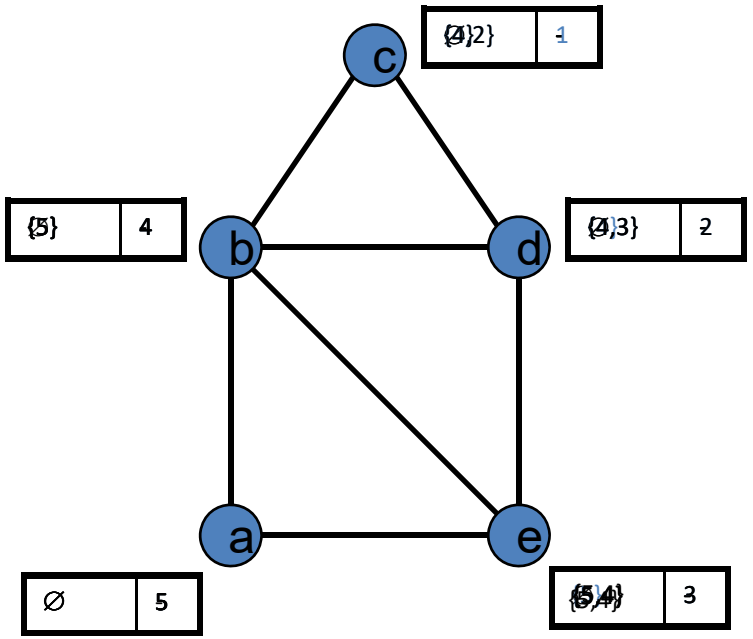
June 8, 2017

Algorithm LEXBFS

1.	Input: Graph G=(V,E) Output: Perfect Elimination Scheme σ if G is chordal
2.	Assign label ∅ to each vertex
3.	FOR i=n TO 1 STEP -1 DO
4.	Pick an unnumbered vertex v with largest label
5.	σ(i) = v
6.	FOR each unnumbered vertex w∈N(v) DO add i to label(w)
7.	RETURN σ

Lexicographic Breadth-first Search (1)

- LEXBFS
  - Select vertex with lexicographic largest label
  - Add order to label of unnumbered adjacent vertices



Step 0: [.,.,.,.,.]

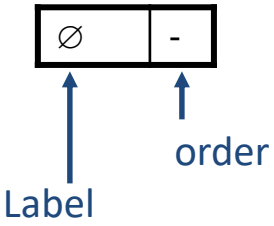
Step 1: [.,.,.,.,a]

Step 2: [.,.,.,b,a]

Step 3: [.,.,e,b,a]

Step 4: [.,d,e,b,a]

Step 5: [c,d,e,b,a]



Algorithm PERFECT

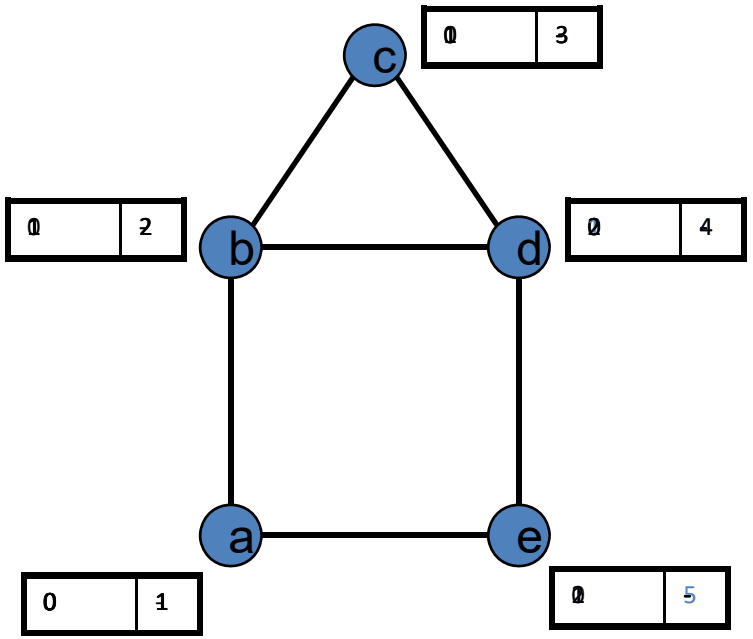
1.	Input: Graph G=(V,E), ordering σ Output: „true“ if σ is a PES and „false“ otherwise
2.	FOR all vertices v DO A(v) := ∅
3.	FOR i=1 TO n-1 DO
4.	v := σ(i);
5.	X := { x∈N(v): $\sigma^{-1}(v) < \sigma^{-1}(x)$ };
6.	IF X=∅ THEN go to 9;
7.	u := σ(min{$\sigma^{-1}(x)$:x∈X})
8.	Concatenate X-{u} to A(u);
9.	IF A(v) – N(v) ≠∅ THEN
10.	RETURN „false“;
11.	RETURN „true“;

Algorithm MCS

1.	Input: Graph G=(V,E) Output: Perfect Elimination Scheme σ if G is chordal
2.	Assign label 0 to each vertex
3.	FOR i=n TO 1 STEP -1 DO
4.	Pick an unnumbered vertex v with highest label
5.	σ(i) = v
6.	FOR each unnumbered vertex w∈N(v) DO add 1 to label(w)
7.	RETURN σ

Maximum Cardinality Search

- MCS
  - Repeatedly select vertex with largest number of labeled neighbors



Step 0: [.,.,.,.,.]

Step 1: [.,.,.,.,a]

Step 2: [.,.,.,b,a]

Step 3: [.,.,c,b,a]

Step 4: [.,d,c,b,a]

Step 5: [e,d,c,b,a]

Algorithm CLIQUES

1.	Input: Graph G=(V,E), perfect elimination scheme σ Output: All maximal cliques and the chromatic number $\chi(G)$
2.	χ:=1; FOR all vertices v DO S(v) := 0
3.	FOR i=1 TO n-1 DO
4.	v := σ(i);
5.	IF N(v) = ∅ THEN go to 3
6.	X := { x∈N(v): $\sigma^{-1}(v) < \sigma^{-1}(x)$ };
7.	IF X = ∅ THEN go to 3;
8.	u := σ(min{$\sigma^{-1}(x)$:x∈X});
9.	S(u) := max(S(u), |X|-1);
10.	IF S(v) < |X| THEN
11.	PRINT {v}∪X;
12.	χ:= max(χ, 1+|X|);
13.	PRINT „The chromatic number is “, χ;