## Mathematics and Gambling

**Ingo Althöfer**
**Friedrich-Schiller-Universität Jena**

What is the distance between Mathematics and gambling? For someone who studied Mathematics in Bielefeld during the 1980's, it is not a large one. We tell four stories of betting and gambling in math.

* Erdős Prizes (100 Dollars and more)
* A stage on the way to solving the board game "Mühle" (10 vs 100 DM)
* The Eternity Puzzle (1 Million British Pounds)
* Gerd Opfer's attempt to solve the Collatz problem (100 vs 10,000 Euro)

Finally, the audience gets a prize puzzle on a combinatorial problem with them (three levels: 500 Euro; 1,000 Euro; 2,500 Euro).

The speaker believes in a variant of Friedrich Schiller's old motto:
**Ein Mathematiker ist nur da ganz Mathematiker, wo er auch wettet.**
(A mathematician is a true mathematician only when he is gambling)

## Finding Defectives by Random Docking and Moving

**Peter Damaschke**
**Chalmers University, Göteborg**

Proteins that repair defectives on the DNA seem to apply some form of group testing on a line. They send signals to each other, and a signal is received if and only if no defective is between sender and recipient. Depending on the signals received, proteins move on the DNA until they hit a defective, or they leave. However the points where the proteins arrive are random. Inspired by this scenario we study a model where robots are searching for defectives on a line. We do not claim that the model accurately reflects the situation in the living cell, rather, we study which abilities of the robots enable which search times. While a group testing strategy that can select the tested intervals would trivially find defectives in logarithmic time, we show that random test points and deterministic moves still find defectives in a time proportional to the square root of the length.

## Worm Colorings of Planar Graphs

**Stanislav Jendrol'**
**Pavol Jozef Safarik University, Košice**

(joint work with Július Czap and Juraj Valiska)

Given three planar graphs $F$, $H$, and $G$. An $(F, H)$-WORM coloring of $G$ is a vertex coloring such that no subgraph isomorphic to $F$ is rainbow and no subgraph isomorphic to $H$ is monochromatic. If $G$ has at least one $(F, H)$-WORM coloring, then $W^{-}_{F,H}(G)$ denotes the minimum number of colors in an $(F, H)$-WORM coloring of $G$. We show that

a) $W^{-}_{F,H}(G) \leq 2$ if $|V(F)| \geq 3$ and $H$ contains a cycle,

b) $W^{-}_{F,H}(G) \leq 3$ if $|V(F)| \geq 4$ and $H$ is a forest with $\Delta(H) \geq 3$,

c) $W^{-}_{F,H}(G) \leq 4$ if $|V(F)| \geq 5$ and $H$ is a forest with $1 \leq \Delta(H) \leq 2$.

We also discuss the remaining cases. The cases when both $F$ and $H$ are nontrivial paths are more complicated; therefore we consider a relaxation of the original problem. Among others, we prove that any 3-connected plane graph (resp. outerplane graph) admits a 2-coloring such that no facial path on five (resp. four) vertices is monochromatic.

## Representations of Posets, Groups, Monoids and Categories

**Jaroslav Nešetřil**
**Charles University, Prague**

Classical results imply that every group (monoid, category) can be represented as the group (monoid, category) of all isomorphisms (endomorphisms, homomorphisms) of a particular graph (or of a class of graphs). While all partial orders may be represented even by oriented paths (and e.g. by outerplanar graphs), for groups, monoids and categories this is not possible. In the context of the sparse hierarchy we determine complexity of these problems in perhaps surprising exactness. This is a recent joint work with Jan Hubicka (Prague) and Patrice Ossona de Mendez (Paris and Prague).

## Burning a Graph

**Dieter Rautenbach**
**Universität Ulm**

Motivated by a graph theoretic process intended to measure the speed of the spread of contagion in a graph, Bonato et al. [Burning a Graph as a Model of Social Contagion, Lecture Notes in Computer Science 8882 (2014) 13-22] define the burning number $b(G)$ of a graph $G$ as the smallest integer $k$ for which there are vertices $x_1, \ldots, x_k$ such that for every vertex $u$ of $G$, there is some $i \in \{1, \ldots, k\}$ with $\text{dist}_G(u, x_i) \leq k - i$, and $\text{dist}_G(x_i, x_j) \geq j - i$ for every $i, j \in \{1, \ldots, k\}$.

For a connected graph $G$ of order $n$, they prove $b(G) \leq 2\lceil\sqrt{n}\rceil - 1$, and conjecture $b(G) \leq \lceil\sqrt{n}\rceil$. We show that $b(G) \leq \sqrt{\frac{32}{19} \cdot \frac{n}{1-\epsilon}} + \sqrt{\frac{27}{19\epsilon}}$ and $b(G) \leq \sqrt{\frac{12n}{7}} + 3 \approx 1.309\sqrt{n} + 3$ for every connected graph $G$ of order $n$ and every $0 < \epsilon < 1$. For a tree $T$ of order $n$ with $n_2$ vertices of degree 2, and $n_{\geq 3}$ vertices of degree at least 3, we show $b(T) \leq \left\lceil\sqrt{(n + n_2) + \frac{1}{4}} + \frac{1}{2}\right\rceil$ and $b(T) \leq \lceil\sqrt{n}\rceil + n_{\geq 3}$. Finally, we show that the problem of deciding whether $b(G) \leq k$ for a given graph $G$ and a given integer $k$ is NP-complete even when restricting the graphs $G$ to either the unions of paths or to trees that have exactly one vertex of degree at least 3.

The presented results are joint work with Stephane Bessy.

## The Partially Disjoint Paths Problem

**Alexander Schrijver**
**University of Amsterdam and CWI Amsterdam**

The partially disjoint paths problem asks for paths between given pairs of terminals, while certain prescribed pairs of paths are required to be disjoint. With the help of combinatorial group theory, we show that, for any fixed number of terminals, this problem can be solved in polynomial time for planar directed graphs. We also discuss related problems. No specific pre-knowledge is assumed.

## Some extremal problems in graph theory

**Zsolt Tuza**
**Alfréd Rényi Institute of Mathematics, Budapest, and University of Pannonia, Veszprém**

Starting from my joint works with Eberhard, I discuss some old and new problems in extremal graph theory.

## Graph Fill-In, Elimination Ordering, Nested Dissection and Contraction Hierarchies

**Dorothea Wagner**
**Karlsruhe Institute of Technology**

Graph fill-in, chordal graph completion, elimination ordering, separators, nested dissection orders and tree-width are only some examples of classical graph concepts that are related in manifold ways. This talk shows how contraction hierarchies, a successful approach to speed up Dijkstras algorithm for shortest paths fits into this series of graph concepts. As theoretical consequence of this insight, a guarantee for the search space required by Dijkstras algorithm combined with contraction hierarchies can be proved. On the other hand, the use of nested dissection leads to a very practicable variant of contraction hierarchies that can be applied in scenarios where edge lengths often change.

## Cannons at Sparrows

**Günter M. Ziegler**
**Freie Universität Berlin**

The story told in this lecture starts with an innocuous little geometry problem, posed in a September 2006 blog entry by R. Nandakumar, an engineer from Calcutta, India: "Can you cut every polygon into a prescribed number of convex pieces that have equal area and equal perimeter?" This little problem is a "sparrow", tantalizing, not as easy as one could perhaps expect, and *Recreational Mathematics*: of no practical use.

I will sketch, however, how this little problem connects to very serious mathematics, including Computational Geometry: For the modelling of this problem we employ insights from a key area of *Applied Mathematics*, the Theory of Optimal Transportation, which leads to weighted Voronoi diagrams with prescribed areas. This will set up the stage for application of a major tool from *Very Pure Mathematics*, known as Equivariant Obstruction Theory. This is a "cannon", and we'll have fun with shooting it at the sparrow.

On the way to a solution, combinatorial properties of the permutahedron turn out to be essential. These will, at the end of the story, lead us back to India, with some time travel 100 years into the past: For the last step in our (partial) solution of the sparrows problem we need a simple divisibility property for the numbers in Pascals triangle, which was first observed by Balak Ram, in Madras 1909.

But even if the existence problem is solved, the Computational Geometry problem is not: If the solution exists, how do you find one? This problem will be left to you. Instead, I will comment on the strained relationship between cannons and sparrows, and to this avail quote a poem by Hans Magnus Enzensberger.