Lehrstuhl II für Mathematik http://www.math2.rwth-aachen.de
Benedikt Annweiler, M.Sc. **WS 2017/18**

Mathematics (for BME)
Problem Sheet 11

Problem 1: Solve the following differential equations:

a) $2xyy' - y^2 + 1 = 0$ with $y(1) = -3$

b) $(x+1)y' = x^2(y+1)$ with $y(1) = 1$

c) $4yy' = (1+2y^2)(3x^2 + \frac{1}{x})$ with $y(1) = -1$

Problem 2: Solve the following differential equations:

a) $x^2y' + 2y = 4e^{-\frac{2}{x}}$ with $(2, \frac{2}{e})$

b) $x^2y' + 2y = e^{\frac{2}{x}}$ with $x_0 \neq 0$

c) $y' = 3\cos x - y\cos x$ with $(0, 2)$

Problem 3: Solve the following non linear differential equations:

a) $y' - xy = e^{-x^2}y^3$ with $y(x_0) = y_0 < 0$

b) $x^2y' + 8y = 4e^{\frac{2}{x}}y^{\frac{3}{4}}$ with $y(1) = 0$

c) $y' = y^2 + 1 - x^2$

Problem 4: Solve the following systems of differential equations:

a) $y' = Ay$ with $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

b) $y' = Ay$ with $A = \begin{pmatrix} -8 & 3 \\ -18 & 7 \end{pmatrix}$

Hint: Even though A in part *b*) is not symmetric you may still use the theorem from the lecture.

Problem 5: Prove the following: If $\lambda \in \mathbb{R}$ is an eigenvalue of A with eigenvector $v \in \mathbb{R}^n \setminus \{0\}$, then a solution to the sytem $y' = Ay$ is given by

$$y = ve^{\lambda x}.$$