
Mathematics (for BME) Problem Sheet 10

Problem 1: Differentiate the following functions $F(x)$:

a) $\int_a^x t^2 - e^{t^2} \log t - 2 \, dt$

b) $\int_a^{x^3} \frac{1}{1 + \sin^2 t} \, dt$

c) $\int_a^b \frac{x}{1 + \sin^2 t} \, dt$

d) $\int_a^{x^3} \sin^3 t \, dt$

e) $\int_a^{g(x)} \frac{1}{1 + \sin^2 t} \, dt$ with $g(x) = \int_a^{x^2} \frac{1}{1 + \sin^2 t} \, dt$

Hint: You do not need to solve the integrals. Leave them in the solution.

Problem 2: Determine the area, that is enclosed by the the functions f and g :

a) $f(x) = x^2, g(x) = \frac{x^2}{2} + 2$

b) $f(x) = x^2, g(x) = -x^2, -1 < x < 1$

c) $f(x) = x^2, g(x) = x^2 - 2x + 4, x > 0$

Problem 3: Evaluate the following indefinite integrals:

a) $\int x^5 + \frac{1}{3}x^3 - 2x \, dx$

b) $\int \log x \, dx$

c) $\int x e^x \, dx$

d) $\int \cos(x)^2 \, dx$

e) $\int \frac{x^2}{\sqrt{1 - 5x^3}} \, dx$

f) $\int \sqrt{x} \log(x) \, dx$

g) $\int \frac{\log x}{x} \, dx$

h) $\int \log(x)^2 \, dx$

i) $\int e^x \sin(e^x) \, dx$

j) $\int e^x e^{e^x} \, dx$

Problem 4: Determine the following with integration by substitution:

$$\int_a^b \log\left(1 + \frac{x}{2}\right) dx \text{ with } a, b > -2$$

Problem 5: Determine if the following improper integrals converge or diverge:

a) $\int_{-\infty}^{\infty} x^3 dx$

b) $\int_{-\infty}^{\infty} e^{-2x} dx$

c) $\int_1^{\infty} \frac{\cos x \sin x}{x^2 + 5x + 1} dx$

d) $\int_0^1 \log x dx$