Mathematics (for BME)
Problem Sheet 9

Problem 1: Consider the following function:

$$f : \mathbb{R} \longrightarrow \mathbb{R} : x \longmapsto 10x^6 + 24x^5 + 15x^4.$$

Determine all minima and maxima of f.

Problem 2: Evaluate the following function limits:

a) $\lim\limits_{x \to 0} \frac{\sin(x)}{x}$

b) $\lim\limits_{x \to 0} \frac{\cos(x)^2 - 1}{x^2}$

c) $\lim\limits_{x \to 0} x \cdot \log(x)$

d) $\lim\limits_{x \to \infty} \frac{x^2}{2^x}$

Problem 3: Evaluate the following function limits:

a) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\tan(5x)}{\tan(x)}$

b) $\lim\limits_{x \to \infty} \sqrt{1 + x^2} \cdot \sin(\frac{1}{x})$

Problem 4: Determine the standard form of the following Taylor polynomials:

a) $T_{3,0}(x)$ for $\exp(x)$

b) $T_{2,1}(x)$ for $\log(x)$

c) $T_{3,0}(x)$ for $\frac{x^2 + 3}{x^2 - 3}$

Problem 5: For $|x| \leq \delta$ with $\delta \in (0,1)$, δ small, the function $(1 + x)^q$ is often approximated by $1 + qx$ with $q \in \mathbb{Q}$. Determine the maximal error of this approximation for $|x| \leq 10^{-2}$ and

a) $q = \frac{1}{2}$,

b) $q = -\frac{1}{2}$.

Problem 6: Prove the following inequations using the Mean Value Theorem:

a) $\sin(x) \leq x$ for all $x \geq 0$

b) $\log(x + 1) \geq \frac{x}{1+x}$ for all $x > 0$