Lehrstuhl II für Mathematik http://www.math2.rwth-aachen.de
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Mathematics (for BME)
Problem Sheet 8

Problem 1: Consider the following choices of $f(x)$:

a) $f(x) = -|x-1|$

b) $f(x) = \frac{x^2-3x+2}{x-2}$

c) $f(x) = \frac{x^2+3x+2}{x-2}$

d) $f(x) = \frac{x^3-7x^2+16x-12}{|x^2+x-6|}$

For each of them solve the following problems:

i) Determine the greatest domain $D \subseteq \mathbb{R}$, such that

$$f : D \longrightarrow \mathbb{R} : x \longmapsto f(x)$$

is well-defined.

ii) Show that f is continuous on that D.

iii) For each $x_0 \in \mathbb{R} \setminus D$ check if f may be continuously extended at x_0. In this case, find the number $y_0 \in \mathbb{R}$ for which $f(x_0) := y_0$ defines a continuous extension.

Problem 2: We consider the following function with two variables:

$$f : \mathbb{R}^2 \setminus \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} \longrightarrow \mathbb{R} : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \frac{2xy}{x^2+y^2}.$$

Show that f cannot be continuously extended at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Problem 3: Use the Intermediate Value Theorem to prove that

$$f : \mathbb{R} \longrightarrow \mathbb{R} : x \longmapsto x^5 - \sin(x) \cdot \exp(2-x^2) + 5$$

has a zero in $[-\frac{\pi}{2}, -1]$. (Hint: $\pi \in [3,4]$)

Problem 4: Compute the radius of convergence for the following power series:

a) $\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

b) $\sum_{k=0}^{\infty} \frac{3^k}{\sqrt{(4k+5)5^k}} x^k$

c) $\sum_{k=0}^{\infty} \frac{2^k}{2k+1} x^k$

d) $\sum_{k=1}^{\infty} 2^{k^2} x^k$

Problem 5: Determine the domain of the following functions $f(x)$. Derive $f(x)$ and determine the points in which $f'(x)$ is not defined:

a) $f(x) = \sqrt{1 + \sin x \cos x}$

b) $f(x) = \sqrt{a^2 + x^2}$, $a \in \mathbb{R} \setminus \{0\}$

c) $f(x) = \frac{3x-6}{x^2-4x+5}$

d) $f(x) = \sqrt[5]{(5x-2)^4}$

e) $f(x) = (x-1) \cdot |x-1|$

f) $f(x) = \frac{x^2|x+1|}{|x-2|}$

g) $f(x) = \frac{\sin x}{x(1-\cos x)}$

h) $f(x) = x^2 \sin(x-2)$

Problem 6: Determine

a) $f'(x)$ for $f(x) = \frac{1}{1-x}$

b) $f''(x)$ for $f(x) = \frac{1}{1-x}$

c) $f'''(x)$ for $f(x) = \frac{1}{1-x}$

d) $f'(x)$ for $f(x) = \log(x^2+1)$

e) $f''(x)$ for $f(x) = \log(x^2+1)$

f) $f'(x)$ for $f(x) = \sin(\exp(x^2) + x)$

g) $f'(x)$ for $f(x) = \log(x)^{\sqrt{x}}, x > 0$

h) $f^{(n)}(x), n \in \mathbb{N}$, for $f(x) = \sin(x)\cos(x)$

(Hint: You are not required to prove that the respective choices of $f(x)$ define functions which can be differentiated sufficiently often.)

Problem 7: Show the following with the help of the power series:

a) $\sin' x = \cos x$

b) $\cos' x = -\sin x$