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## Mathematics (for BME)

### Problem Sheet 7

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**Problem 1:** Prove that

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$$

by means of the definition of limits of sequences.

**Problem 2:** Evaluate the following limits of sequences:

a)  $\lim_{n \rightarrow \infty} \sqrt{\frac{5}{n}}$

b)  $\lim_{n \rightarrow \infty} 2^{-n} + 2^{-\frac{1}{n}}$

c)  $\lim_{n \rightarrow \infty} n \cdot (1 + (-1)^n)$

d)  $\lim_{n \rightarrow \infty} \frac{4n^3}{n^4 + n^2 + 1}$

e)  $\lim_{n \rightarrow \infty} \frac{(2n+1)(3n-1)}{(4n+1)(2n-3)}$

f)  $\lim_{n \rightarrow \infty} \frac{3n^3 - 1}{4n^2 + 2}$

g)  $\lim_{n \rightarrow \infty} \frac{(-1)^n}{n}$

h)  $\lim_{n \rightarrow \infty} \frac{(-1)^{2n+1}}{2n+1}$

i)  $\lim_{n \rightarrow \infty} (2n)^{\frac{1}{n}}$

j)  $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$

k)  $\lim_{n \rightarrow \infty} \frac{n^2}{4^n}$

l)  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+5}} \binom{n}{2}$

m)  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n^2}\right)^n$

n)  $\lim_{n \rightarrow \infty} \cos\left(\frac{\pi}{2}n\right)$

o)  $\lim_{n \rightarrow \infty} (-1)^n$

**Problem 3:** Determine if the following sequences converge and calculate the limit of the sequence in case it converges.

a)  $a_1 = 1, a_{n+1} = a_n(2 - a_n)$

b)  $a_1 = \sqrt{2}, a_{n+1} = \sqrt{2 + a_n}$

c)  $a_1 = 3, a_{n+1} = \frac{7+3a_n}{3+a_n}$

d)  $a_1 = 2, a_{n+1} = \frac{a_n^2}{4} (\sqrt{a_n^2 + 4} - a_n)$

**Problem 4:** Show that the following series converges to its given limit:

$$\sum_{k=2}^{\infty} \frac{1}{(k-1)k} = 1$$

**Problem 5:** Show that the following series converges to its given limit:

$$\sum_{k=1}^{\infty} \frac{2 + (-1)^k}{3^k} = \frac{15}{4}$$

**Problem 6:** Check the series

$$\sum_{k=1}^{\infty} a_k$$

for convergence or divergence if

a)  $a_k = \frac{1}{\sqrt{k}}$

b)  $a_k = \frac{1}{2k}$

c)  $a_k = \left(-\frac{2}{3}\right)^k$

d)  $a_k = \frac{1}{k} - \frac{1}{k^2}$

e)  $a_k = \left(\frac{k}{k+1}\right)^k$

f)  $a_k = \frac{k^k}{2^k \cdot k!}$

g)  $a_k = \frac{\cos(k\pi)}{23\sqrt{k^2+4k+2}}$

h)  $a_k = \frac{k^4}{k!}$

i)  $a_k = \left(\frac{k+1}{2k+1}\right)^k$

j)  $a_k = (\sqrt[k]{k} - 3)^k$

k)  $a_k = (-1)^k \left(\frac{1}{\sqrt{k}} + 1\right)$

for all  $k \in \mathbb{N}$ .