Lehrstuhl II für Mathematik http://www.math2.rwth-aachen.de
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Mathematics (for BME)
Problem Sheet 6

Problem 1: Check each of the following matrices for invertibility and find its inverse in case it exists:

$$A := \begin{pmatrix} 2 & -2 \\ 0 & 3 \end{pmatrix} \qquad B := \begin{pmatrix} 2 & 3 & -1 \\ 3 & 1 & -1 \\ 12 & 11 & -5 \end{pmatrix} \qquad C := \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 \\ -1 & 0 & -1 & \frac{1}{2} \end{pmatrix}$$

Problem 2: The following matrices are only combinations of the basic transformations R_α, S and D_λ. Determine what they do:

$$A := \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad B := \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad C := \begin{pmatrix} 4 & -4 \\ 4 & 4 \end{pmatrix}$$

Problem 3: Calculate the determinant for the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 3 \\ 0 & -1 & 0 & 2 \\ 4 & 2 & 3 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}$$

Problem 4: Find the eigenvalues and a basis for the respective characteristic spaces for each of the following matrices over $\mathbb{C}$. Determine the multiplicity as well as geometric multiplicity of all eigenvalues.

$$A := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad B := \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad C := \begin{pmatrix} 0 & 4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Problem 5: We consider the following three stage population model of some species. We have the three age stages childhood, maturity and seniority. For those three stages we have a survival rate and an offspring rate. For those in the childhood stage, only $\frac{2}{7}$ survive and reach maturity and they do not produce any offsprings. The mature ones produce 3 offsprings and have a survivalrate of 50%. The ones in the seniority stage produce one offspring but do not survive any longer. Find a distribution into the three stages of age such that the ratio between the three stages does not change over time or to the next cycle. Does the population grow or shrink for those ratios?

Problem 6: We consider a population of insects with two age states: *young* and *mature*. One quarter of the young insects turn into mature insects after one period. The other three quarters die before they reach maturity. Only mature insects produce offspring. More precisely, the number of young insects in the next period equals two times the number of mature insects in the present period. One half of the present mature insects survive and are still alive in the next period, the others die.
Let v_1 and v_2 denote the initial number of young and mature insects respectively.

a) Find a matrix $A \in \mathbb{R}^{2\times 2}$ such that $A \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ denotes the number of young and mature insects after one period.

b) Find a matrix $B \in \mathbb{R}^{2\times 2}$ such that $B \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ denotes the number of young and mature insects after three periods.
Calculate those numbers for $v_1 = 440$ and $v_2 = 620$.

c) Assume there are 1020 young and 2000 mature insects in the present period. What were the numbers of young and mature insects one period before?

d) Is there a choice of v_1 and v_2 such that the numbers of young and mature insects stay constant? In case there is, find such a choice.