Lehrstuhl II für Mathematik http://www.math2.rwth-aachen.de
Benedikt Annweiler, M.Sc. **WS 2017/18**

Mathematics (for BME)
Problem Sheet 5

Problem 1: Prove that a set of n non-zero vectors $\{v_1, ..., v_n\} \subseteq \mathbb{R}^m \setminus \{0\}$ is linearly independent, if all vectors are *mutually orthogonal*, i. e. if $i \neq j \Rightarrow v_i \cdot v_j = 0$ holds for all $i, j \in \{1, ..., n\}$.

Problem 2: Let $b = \begin{pmatrix} \sqrt{2} \\ 0 \\ -1 \end{pmatrix}$ be a vector. Determine the set of all vectors $v \in \mathbb{R}^3$ that satisfy the following 3 conditions:

i) v has length 2,

ii) $\measuredangle(v, e_2) = \frac{\pi}{3}$,

iii) v is orthogonal to b.

Problem 3: Let $b = \begin{pmatrix} \sqrt{2} \\ 0 \\ -1 \end{pmatrix}$ be a vector. Determine the set of all vectors $v \in \mathbb{R}^3$ that satisfy the following 3 conditions:

i) v is orthogonal to b,

ii) v is orthogonal to $e_1 + e_2$,

iii) v has length $\sqrt{8}$.

Problem 4: Determine if the following maps are linear. If a map is linear, then note its associated matrix.

a) $f : \mathbb{R}^3 \to \mathbb{R} : f(x, y, z) = x + y + 3$,

b) $f : \mathbb{R}^2 \to \mathbb{R} : f(x, y) = 2x + y$,

c) $f : \mathbb{R}^3 \to \mathbb{R}^2 : f(x, y, z) = \begin{pmatrix} x^2 \\ y - z \end{pmatrix}$,

d) $f : \mathbb{R}^2 \to \mathbb{R}^4 : f(x,y) = \begin{pmatrix} x \\ 0 \\ 2x \\ xy \end{pmatrix}$,

e) $f : \mathbb{R}^3 \to \mathbb{R}^2 : f(x,y,z) = \begin{pmatrix} 5xy - 3xz \\ 2xy - 2yz \end{pmatrix}$,

f) $f : \mathbb{R}^3 \to \mathbb{R}^3 : f(x,y,z) = \begin{pmatrix} 2x^2 + \frac{1}{2}xy - (y+4x)\frac{1}{2}x + z \\ 2y + 3(x-1) + 3 \\ 3z^3 + 6(-z^2 - 4y)\frac{1}{2}z + 12yz \end{pmatrix}$,

g) $f : \mathbb{R} \to \mathbb{R}^5 : f(x) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T$,

Problem 5: Determine the set of all solutions and the defect for the following systems of linear equations:

i)

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 - x_3 + 2x_4 &= 5 \\ -2x_1 + 4x_2 + 2x_4 &= -2 \end{aligned}$$

ii)

$$\begin{aligned} x_1 + 3x_2 - 4x_3 + 3x_4 &= 9 \\ 3x_1 + 9x_2 - 2x_3 - 11x_4 &= -3 \\ 4x_1 + 12x_2 - 6x_3 - 8x_4 &= 6 \\ 2x_1 + 6x_2 + 2x_3 - 14x_4 &= -12 \end{aligned}$$

Problem 6: Determine if M is a subspace and/or an affine subspace.

a) $M = \{(x,y,z) \in \mathbb{R}^3 |\ 3x + 4y + 3z = 4\}$

b) $M = \{(x,y,z) \in \mathbb{R}^3 |\ 3x^2 + 5y^2 + 3z^2 = 0\}$

c) $M = \{(x,y,z) \in \mathbb{R}^3 |\ 5x + 9y \leq 0\}$

d) $M = \{(x,y) \in \mathbb{R}^2 |\ x + y = \pi\}$

e) $M = \{(x,y) \in \mathbb{R}^2 |\ x + y = 0\}$

f) $M = \{p + x^4 |\ p$ is a polynomial with degree less or equal to $2\}$

g) $M = \{p + x^4 |\ p$ is a polynomial with degree equal to $2\}$