
Mathematics (for BME) Problem Sheet 4

Problem 1: Decompose (i. e. factorize) the following polynomials into irreducible factors over \mathbb{C} :

a) $x^3 - x^2 - x - 2$

b) $x^2 - x + 1$

c) $x^3 + 3x^2 - 4$

d) $x^5 - x^3$

e) $x^2 + 1$

e) $x^4 + 6x^2 + 5$

Problem 2: Determine the polynomial $p(x)$ of degree n for the following points in the plane:

a) $n = 4, (0,0), (1,6), (2,40), (-1,-2), (-2,0)$

b) $n = 3, (0,1), (1,1), (2,7), (-1,1)$

Problem 3: The following matrices are given:

$$A := \begin{pmatrix} 1 & 3 & 2 & 4 \\ 2 & 3 & -2 & -2 \end{pmatrix}, B := \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}^T,$$

$$C := \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, D := \begin{pmatrix} -2 & 0 \\ 1 & 1 \\ 3 & -7 \end{pmatrix}.$$

Determine the following matrices: $A^2, A \cdot B, A \cdot B^T, A \cdot C, C \cdot A, B \cdot B^T, B^T \cdot B, C^2, C \cdot D, D \cdot C$.

Which of those matrix products are not defined?

Problem 4: Solve the system $Ax = b$ of linear equations for the following A and b :

a) $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 3 \\ 3 & -1 & -4 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

b) $A = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 3 \\ 3 & -1 & -4 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$c) A = \begin{pmatrix} 2 & 1 & 3 & 4 \\ 1 & 4 & 2 & 2 \\ 2 & 3 & -1 & -1 \\ 5 & -1 & 1 & 4 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

Problem 5: The following sets of vectors are given in the \mathbb{R}^n . Determine if those sets are linearly independent.

$$a) \left\{ \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \right\}$$

$$b) \left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \right\}$$

$$c) \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix} \right\}$$

$$d) \left\{ \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -2 \end{pmatrix} \right\}$$

Determine the dimension of the vector space created by those sets of vectors.