Lehrstuhl II für Mathematik http://www.math2.rwth-aachen.de
Benedikt Annweiler, M.Sc. **WS 2017/18**

Mathematics (for BME)
Problem Sheet 3

some important values for $e^{i\varphi}$:

φ	$e^{i\varphi}$
0	1
$\pi/12$	$(\sqrt{6}+\sqrt{2})/4+i(\sqrt{6}-\sqrt{2})/4$
$\pi/6$	$\sqrt{3}/2+i/2$
$\pi/4$	$1/\sqrt{2}+i/\sqrt{2}$
$\pi/3$	$1/2+i\sqrt{3}/2$
$5\pi/12$	$(\sqrt{6}-\sqrt{2})/4+i(\sqrt{6}+\sqrt{2})/4$
$\pi/2$	i

some properties of $e^{i\varphi}$:

$e^{i(\varphi+2\pi)}=e^{i\varphi}$	$e^{-i\varphi}=\overline{e^{i\varphi}}$
$e^{i(\varphi+\pi)}=-e^{i\varphi}$	$e^{i(\varphi+\pi/2)}=ie^{i\varphi}$

Problem 1: Determine $Re(z)$, $Im(z)$ and $|z|$ for the following complex numbers:
a) $\frac{1-i}{1-2i}z=\frac{2+2i}{1+3i}$ b) $(2+i)^2+7-3i$
c) $z=\frac{i+3}{2i-4}$ d) $\left(\frac{1-i}{2+3i}-\frac{6+2i}{1+i}\right)z=\frac{3-i}{3+i}$

Problem 2: Determine z:

a) $2z+3i\overline{z}-5\overline{z}=5(i-1)$

b) $-3z+\overline{z}-2i\overline{z}=2i$

Problem 3: Find the polar coordinates of the following complex numbers:

a) $1+i$ b) $\frac{-1}{2}+\frac{\sqrt{3}}{2}i$

c) 4 d) -4

e) $\sqrt{3}-i$ f) $-5i$

g) $-\frac{5}{2}\sqrt{3}-\frac{5}{2}i$ h) $-\frac{3}{2}-\frac{3}{2}\sqrt{3}i$

Take the polar coordinates both for $arg(z)\in[0,2\pi]$ and $arg(z)\in(-\pi,\pi]$. Try to find the polar coordinates of $a)-f)$ **without** the help of the tabular. A calculator might be needed then.

Problem 4: Determine and sketch $M \subset \mathbb{C}$ defined as followed:

a) $3z^2 - 10z\bar{z} + 3\bar{z}^2 + 16 = 0$

b) $Re(z) \leq |z|^2$

c) $|z - i| \leq |z - 2 + i|$ and $|arg(z+2)| < \frac{\pi}{4}$

The argument is in $(-\pi, \pi]$.

Problem 5: Solve the equations for z and draw all roots into the complex plane.

a) $z^3 - 1 = 0$

b) $z^2 = 4i$

c) $z^6 = 1$

d) $z^3 = \frac{1}{\sqrt{2}}(1 - i)$

Problem 6: Determine $Re(z)$ and $Im(z)$.

a) $z = \left(\frac{1-i}{1+i}\right)^{10}$

b) $z = \left(\frac{-1+i\sqrt{3}}{6i}\right)^{1991}$

Problem 7: For the following polynomials $p(x), q(x)$ determine polynomials $a(x), r(x)$ such that $\deg(r) < \deg(q)$ and $p(x) = q(x) \cdot a(x) + r(x)$:

a) $p(x) = x^3 - 2x + 1, \quad q(x) = x^2$

b) $p(x) = x^4 - x^3 + 3x^2 - 2x + 2, \quad q(x) = x^2 - x + 1$

c) $p(x) = x^n + x^{n-1} + ... + x + 1, \quad q(x) = x + 1, \quad n$ odd

Problem 8: Decompose (i. e. factorize) the following polynomials into irreducible factors (with coefficients in $\mathbb{R}$):

a) $x^3 - x^2 - x - 2$

b) $x^2 - x + 1$

c) $x^3 + 3x^2 - 4$

d) $x^5 - x^3$

Those are too many questions to actually solve all in one exercise. The problem sheet is due wednesday but we will probably only do one part per problem.