Lehrstuhl II für Mathematik http://www.math2.rwth-aachen.de
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Mathematics (for BME)
Problem Sheet 2

Problem 1: Prove *Bernoulli's Inequality* using induction:
For all $n \in \mathbb{N}$ and all real $a \geq -1$ we have

$$(a+1)^n \geq na + 1.$$

Problem 2: Prove the *Geometric Sum Formula* using induction:
For all $n \in \mathbb{N}$ and all $q \in \mathbb{R} \setminus \{1\}$ we have

$$\sum_{i=0}^{n} q^i = \frac{q^{n+1}-1}{q-1}.$$

Problem 3: Prove the following formulas for all $n \in \mathbb{N}$ using induction:

i) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

ii) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

iii) $\sum_{k=1}^{2n} (-1)^{k+1} \frac{1}{k} = \sum_{k=1}^{n} \frac{1}{n+k}$

Problem 4: Determine the $n \in \mathbb{N}$ for which the following inequalities hold:

i) $n^2 \leq 2^n$

ii) $3^n < n!$

Problem 5: Proof the following statements:

i) $n^2 + n$ is divisible by 2.

ii) $5^n + 7$ is divisible by 4.

Problem 6: Prove the following using induction:

$$\prod_{k=1}^{n-1} \left(1 + \frac{1}{k}\right)^k = \frac{n^n}{n!}, \ \forall n \in \mathbb{N}, n \geq 2.$$

Problem 7: The inventor of the board game *chess* was granted a wish by the Shah of Persia. He simply asked the Shah for grains of wheat: One grain for the first square of the chess board, another two grains for the second square, another four grains for the third square, and so on, always doubling the number of grains for each square. The Shah laughted at this modest wish and advised his men to bring the grains.
How many grains was the inventor of chess actually asking for?
(Today approximately 10^{16} grains of wheat are produced in the world every year.)

Problem 8: Ten players have to be divided into two teams of equal size. How many ways are there to accomplish this?