
Mathematics (for BME) Problem Sheet 1

Problem 1: Simplify the following terms ($a, b, c > 0, n \in \mathbb{N}$):

a) $\frac{9a^2(2b)^2c^5}{6a^3bc^3}$	b) $2^{n+1} - 2^n$	c) $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$
d) $\frac{\sqrt[3]{a}\sqrt[5]{a}}{\sqrt[4]{a}}$	e) $\frac{1 + \frac{1}{2n}}{1 - \frac{3}{2n}} - \frac{4}{2n - 3}$	f) $(2a + 1)(b - 3) - (2b - 1)(a + 3)$
g) $\frac{(a^{m+3}a^{8m+1})^{9m-4}}{a^{-m^2}}$	h) $\frac{\sqrt{a^2b}}{b\sqrt{a}}$	

Problem 2: Determine the following sets:

a) $\{m \in \mathbb{Z} m^2 < 16\} \cup \{1, 2, 3, 4, 5\}$	b) $\{m \in \mathbb{Z} m^2 < 16\} \cap \{1, 2, 3, 4, 5\}$
c) $\{m \in \mathbb{Z} m^2 < 16\} \setminus \{1, 2, 3, 4, 5\}$	d) $[2, 10] \setminus [8, 10]$
e) $(-3, 3] \cup (1, \infty)$	f) $(-3, 3] \cap (1, \infty)$
g) $(-3, 3] \setminus (1, \infty)$	h) $(1, \infty) \setminus (-3, 3]$

Problem 3: Check each of the following functions in a) to d) for Injectivity / Surjectivity / Bijectivity:

a) $f_1 : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto x^2$	b) $f_2 : \mathbb{R} \rightarrow [0, \infty) : x \mapsto x^2$
c) $f_3 : [0, \infty) \rightarrow \mathbb{R} : x \mapsto x^2$	d) $f_4 : [0, \infty) \rightarrow [0, \infty) : x \mapsto x^2$

Problem 4: Find a suitable set A and two functions $f : A \rightarrow A$ and $g : A \rightarrow A$ such that $g \circ f \neq f \circ g$.

Problem 5: Let A be a non-empty finite set. Describe all functions $f : A \rightarrow A$ which satisfy the property $f \circ f = f$. How many bijective functions f satisfy this property?

Problem 6: Let A and B be finite, non-empty sets and $f : A \rightarrow B$ a function. Let $|A|$ and $|B|$ denote the number of elements in A and B respectively. Prove the following facts:

a) If f is injective then $|A| \leq |B|$.

b) If f is surjective then $|A| \geq |B|$.

c) If f is bijective then $|A| = |B|$.

Problem 7: Find all real numbers $x \in \mathbb{R}$ that satisfy the following equations or inequations:

a) $7x^2 + 19 = 82$

c) $12x^6 + 6x^5 = 0$

e) $\frac{1}{x+1} \leq 2$

b) $-3x^4 + 3x^2 = -6$

d) $\frac{2x+3}{4x-2} = \frac{4x+6}{8x+4}$

f) $\sqrt{x^2-9} \leq \frac{x+3}{2}$