2 Complete Induction

Definition 1 (induction principle). *If $M \subset \mathbb{N}$ has the properties*

(i) $1 \in M$,

(ii) $n \in M$ *implies that* $n + 1 \in M$,

then $M = \mathbb{N}$.

Definition 2 (induction principle). *Let $n_0 \in \mathbb{Z}$ and for $n \in \mathbb{Z}$ with $n \geq n_0$ let $A(n)$ be a proposition (depending on n). If*

(i) $A(n_0)$ *is true,*

(ii) For all $n \in \mathbb{Z}$ with $n \geq n_0$: $A(n) \Rightarrow A(n+1)$,

then $A(n)$ is true for all $n \in \mathbb{Z}$ with $n \geq n_0$. Part (i) is called the basis, *Part (ii) the* inductive step. *The assumption in the inductive step that $A(n)$ holds for some (arbitrary) $n \geq n_0$ is called the* induction hypothesis.

Theorem 3 (geometric sum). *If $n \in \mathbb{N}_0$ and $q \in \mathbb{R}$, then*

$$(1-q)\sum_{k=0}^{n} q^k = 1 - q^{n+1}.$$

Theorem 4 (sum of the first n natural numbers). *If $n \in \mathbb{N}$, then*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Definition 5 (factorial). *For $n \in \mathbb{N}$ we define the* factorial *of n by*

$$n! = \prod_{k=1}^{n} k = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n.$$

Furthermore, we define $0! = 1$.

Theorem 6 (number of permutations). *There are* $n!$ *different possibilities of arranging* n *distinct objects in a sequence (the arragements are called* permutations*).*

Definition 7 (binomial coefficient). *For* $n \in \mathbb{N}_0$ *and* $k \in \mathbb{N}_0$ *with* $k \leq n$*:*

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n \cdot (n-1) \cdot \ldots \cdot (n-k+1)}{k!}.$$

Theorem 8 (number of subsets). *If* M *is a set with* n *elements, the number of subsets of* M *with* k *elements is* $\binom{n}{k}$*.*

Theorem 9 (addition of binomial coefficients). *If* $n \in \mathbb{N}_0$ *and* $k \in \mathbb{N}_0$ *with* $k \leq n+1$*, then*

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$

Theorem 10 (binomial theorem). *If* $n \in \mathbb{N}$ *and* $x, y \in \mathbb{R}$*, then*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$