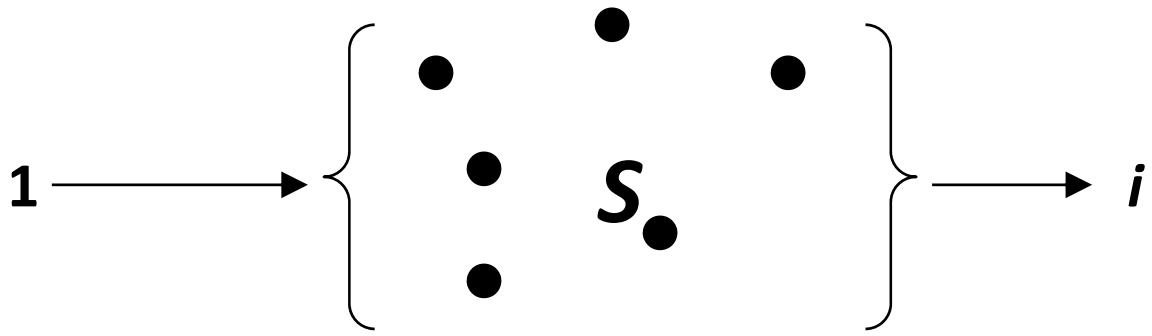
Algorithm Graph Theory: How hard is your combinatorial optimization problem?

Short Course – Lecture 12
June 14, 2017

Slides available at:
https://www.math2.rwth-aachen.de/de/mitarbeiter/koster/agtclemson

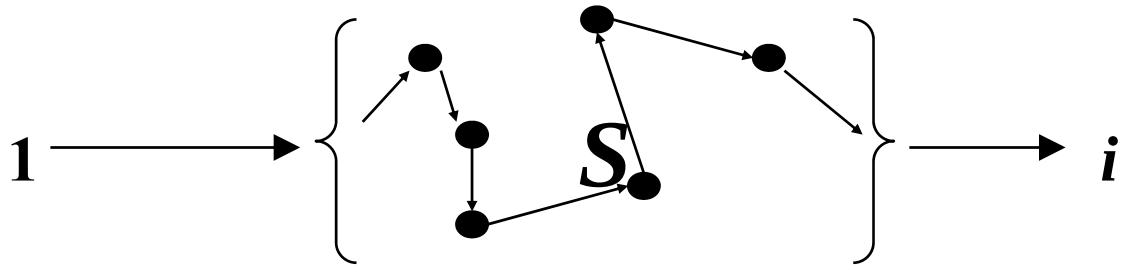
TSP: Dynamic Programming

For any $S \subseteq \{2,3,\ldots,n\}$ and $i \notin S$, let **(S,i)-path** be a path which starts at city 1, visits each city in S exactly once, and no other city, and ends up in city i.



TSP: Dynamic Programming

Let **cost**[*S,i*] stand for the length of the shortest *(S,i)* path



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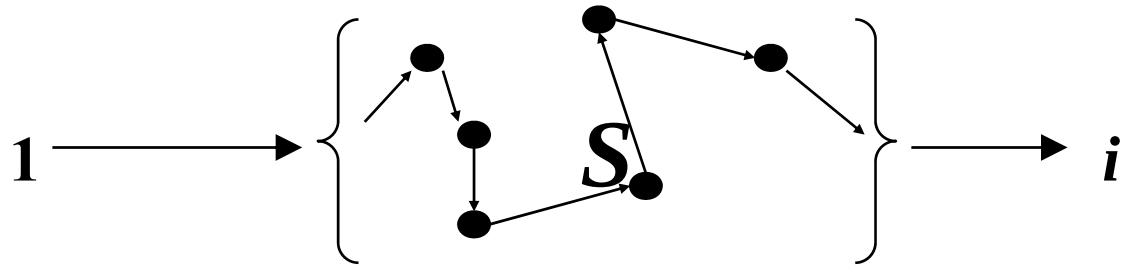
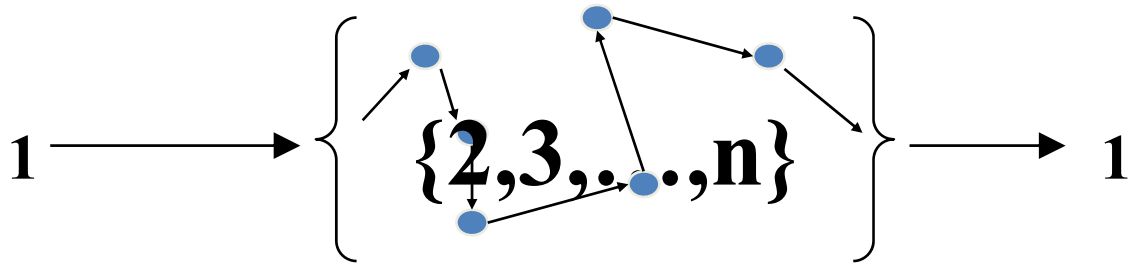


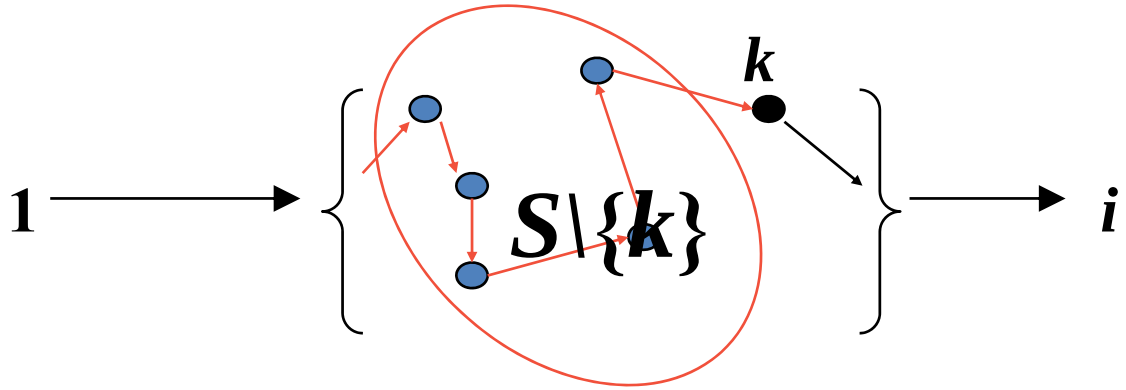
Then the TSP is equivalent to the problem of finding **cost**[{2,3,…,*n*},1]



TSP: Dynamic Programming

The lengths **cost**[*S,i*] have to satisfy the following equations:

$$\text{cost}[S,i]=\min_{k\in S}\{\text{cost}[S\backslash\{k\},k]+c_{ki}\}$$



THE TRAVELLING SALESMAN PROBLEM: DP

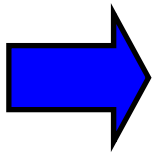
How many entries does the **cost**[*S,i*] table have?

$$(n-1)*(n-2)+\binom{n-1}{2}*(n-3)+\binom{n-1}{3}*(n-4)+\binom{n-1}{4}*(n-5)+\ldots+\binom{n-1}{n-2}*1+1$$

$$=2*\binom{n-1}{2}+3*\binom{n-1}{3}+4*\binom{n-1}{4}+5*\binom{n-1}{5}+\ldots+(n-1)*\binom{n-1}{n-1}+1$$

$$=(n-1)*2^{n-2}-(n-2)$$

n=	5	10	20	30	40	50	100	1000
TSP	29	2296	4980718	7784628196	1.072E+13	1.3792E+16	3.1374E+31	2.67E+303
Knaps B=500	2500	5000	10000	15000	20000	25000	50000	500000



Computational not a feasible approach for larger number of cities